Peter Lee on November 16, 2009

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Recall Verma modules:

$$M_\pm := U(\mathfrak{g}) \otimes_{\mathfrak{g}_\pm} 1_{\mathfrak{g}_\pm} \cong U(\mathfrak{g}_\mp) \otimes_{\mathfrak{g}_\pm} 1_\pm \cong U(\mathfrak{g}_\mp)$$

There are co-products $i_\pm : M_\pm \to M_\pm \otimes M_\pm$

Also, $\mathcal{M}$ is the category with

$$Ob(\mathcal{M}) = \{\text{reps of } \mathfrak{g}\}$$

$$Mor_{\mathcal{M}}(U, V) := Hom_{\mathfrak{g}}(U, V)[[\hbar]]$$

it is a braided monoidal category with $R = e^{\hbar t^{12}/2}$.

<u>Lemma</u> The assignment $1 \to 1_+ \otimes 1_-$ extends $\mathfrak{g}$-invariantly to an isomorphism of $\mathfrak{g}$-modules

$$U(\mathfrak{g}) \xrightarrow[\phi]{\sim} U(\mathfrak{g}_-) \otimes U(\mathfrak{g}_+) = M_+ \otimes M_-$$

We want to define a BA structure on $U(\mathfrak{g})$ (i.e., a new coproduct and unit).

Note that if $g \in U(\mathfrak{g})$ we can view $g \in End(F)$, where $F : \mathcal{M} \to Vect[[\hbar]]$.

Let $F^2 : \mathcal{M} \times \mathcal{M} \to Vect[[\hbar]]$ be $(U, V) \mapsto F(U) \otimes F(V)$.

We need a $\Delta : End(F) \to End(F^2)$.

Recall that there is an iso $\theta : U(\mathfrak{g}) \to End(F)$

If we had natural isomorphisms:

$$I_{UV} : F(U) \times F(V) \xrightarrow{\sim} F(U \otimes V)$$

we could define $g : F(U) \otimes F(V) \circlearrowleft$ by

$$F(U) \otimes F(V) \xrightarrow{I_{UV}} F(U \otimes V) \xrightarrow{g} F(U \otimes V) \xrightarrow{I_{UV}^{-1}} F(U) \otimes F(V)$$

Recall A tensor structure on a functor $F : \mathcal{C} \to \mathcal{C}'$ b/w monoidal categories is a family of natural iso. J_{XY} $\forall X, Y \in \mathcal{C}$,

$$J_{XY} : F(X) \otimes F(Y) \longrightarrow F(X \times Y)$$

s.t. some coherence laws apply.

A tensor structure on $F = \mathrm{Hom}(U(\mathfrak{g}), -)$
$= \mathrm{Hom}(M_+ \otimes M_- \longrightarrow \_\_)$:

Let $V, W \in \mathcal{M}$, $v, w \in F(V), F(W)$ define

$$J_{VW} : \mathrm{Hom}(M_+ \otimes \overset{v}{\hat{M}_-}, V) \otimes \mathrm{Hom}(M_+ \otimes \overset{w}{\hat{M}_-}, W) \to \mathrm{Hom}(M_+ \otimes M_-, V \otimes W)$$

by

$$J_{VW}(v \otimes w) : M_+ \otimes M_- \xrightarrow{i_+ \otimes i_-} (M_+ \otimes M_+) \otimes (M_- \otimes M_-)$$

$$\xrightarrow[\alpha R's]{\Phi's} (M_+ \otimes M_-) \otimes (M_+ \otimes M_-) \xrightarrow{v \otimes w} V \otimes W$$